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ONE-DIMENSIONAL STABILITY OF DISSIPATIVE COUETTE FLOW*

V.L. MAZO and M.S. RUDERMAN

Non-stationary one-dimensional flow, and especially the one-dimensional stability of a stationary Couette flow of a viscous incompressible fluid is considered, taking into account dissipative heat, under the assumption that the viscosity decreases fairly rapidly, e.g., exponentially, as the temperature incrases. It is shown that when the fluid is very viscous, the non-stationary plane problem can be reduced to a non-stationary problem of heat transfer in media with heat sources depending non-linearly on temperature. The dependence of the heat sources on temperature in the latter problem differs substantially for different types of boundary conditions in the initial problem. If a tangential stress is specified at the boundary, then the density of the heat sources will depend on temperature locally (such a problem was studied earlier in /1-6/. When the velocities of the boundary planes are given, the density of heat sources will depend on the temperature distribution as a whole, over the volume.

As regards the stationary flow inspected here for stability, it does not always exist, nor is it unique /7-14/. We can utilize the results of /1, 2, 15-19/ by reducing the problem of the existence and uniqueness of such flow to the stationary problem of the temperature distribution in media with heat sources depending non-linearly on temparture.

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1. Let us suppose that z_* is the transverse coordinate, t_* is the time, T is the temperature, u_* is the velocity, τ_* is the tangential stress, c is the heat capacity, ρ is the density and λ is the thermal conductivity (the last three parameters are assumed constant) and $\varphi_*(T)$ is the fluidity (the inverse dynamic viscosity) depending on the temperature T. We will change to the following dimensionless coordinates: $z = z_*/h$ where h is the half-distance between the two boundary planes, $t = t_*/t_0$ where $t_0 = \rho c h^2/\lambda$; $\vartheta = (T - T_0)/\Delta T$ where T_0 is some characteristic temperature, and ΔT is its characteristic scattering $u = u_*/u_0$ where $u_0^2 = 2\lambda\Delta T\varphi_*(T_0)$; $\tau = \tau_*/\tau_0$ where $\tau_0 = u_0/(\varphi_*(T_0)h)$; $\varphi(\vartheta) = \varphi_*(T)/\varphi_*(T_0)$ and in the case of exponential fluidity we have $\varphi(\vartheta) = e^{\vartheta}$.

The equations describing the flow in question have the following form in dimensionless variables (Pr is the Prandtl number):

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial^2 \vartheta}{\partial z^2} + 2\tau^2 \varphi(\vartheta), \quad \frac{1}{\mathrm{Pr}}\frac{\partial u}{\partial t} = \frac{\partial \tau}{\partial z}, \quad \frac{\partial u}{\partial z} = \varphi(\vartheta)\tau \tag{1.1}$$

We assume that equal constant temperature is specified on both boundary planes

$$z = \pm 1, \quad \vartheta = 0 \tag{1.2}$$

We consider two types of dynamic boundary conditions. In the first case we assume that a constant tangential stress is specified on one of the boundary surfaces

(type 1) $$z = 1, \tau = \tau_1; \; z = 0, u = 0 \tag{1.3}$$

(The second condition is not essential and is chosen so that, in what follows, certain formulas will be found to be identical for both types of boundary conditions). In the second case we assume that the velocities of the boundary planes are given

(type 2) $$z = \pm 1, \quad u = \pm u_1 \tag{1.4}$$

The stationary version of Eq.(1.1) has the form

$$\frac{\partial^2 \bar\vartheta}{\partial z^2} + 2\bar\tau^2 \varphi(\bar\vartheta) = 0, \quad \frac{\partial \bar\tau}{\partial z} = 0, \quad \frac{\partial \bar u}{\partial z} = \varphi(\bar\vartheta)\bar\tau \tag{1.5}$$

According to the second equation of (1.5), $\bar\tau$ is constant. In the case of dynamic boundary conditions of type 1 (1.3) we have

(type 1) $$\bar\tau = \tau_1 \tag{1.6}$$

and in the case of conditions of type 2 (1.4) we average the third equation of (1.5) over the interval from -1 to 1 to obtain

(type 2) $$\bar\tau \langle \varphi(\bar\vartheta) \rangle = u_1 \tag{1.7}$$

where $\langle \ldots \rangle$ denotes averaging.

In this manner we reduce system (1.5) to its first equation supplemented by (1.6) or (1.7). When the value of $\bar\tau$ is given, the first equation of (1.5) is closed and identical to the stationary heat transfer equation in the media with heat sources depending non-linearly on temperature, and this enables us to utilize the already-known restults of /1, 2, 15-19/. These results should be applied to the problem under consideration taking into account the connection between $\bar\tau$ and the dynamic boundary conditions given by relations (1.6) or (1.7).

When the law governing the fluidity is exponential, the stationary problem has the following solution /9, 10, 13-15/:

$$\bar\vartheta = \ln \frac{\mathrm{ch}^2 \sigma}{\mathrm{ch}^2(\sigma z)}, \quad \bar\tau = \frac{\sigma}{\mathrm{ch}\,\sigma}, \quad \bar u = \mathrm{sh}\,\sigma \frac{\mathrm{th}(\sigma z)}{\mathrm{th}\,\sigma} \tag{1.8}$$

where σ is a parameter determined in the case of type 1 boundary conditions from the equation

(type 1) $$\tau_1 = \sigma/\mathrm{ch}\,\sigma \tag{1.9}$$

and in the case of type 2 boundary conditions from the equation

(type 2) $$u_1 = \mathrm{sh}\,\sigma \tag{1.10}$$

Formulas (1.8) together with Eq.(1.9) or (1.10) yield a complete solution of system (1.5) with boundary conditions (1.2)-(1.4). This yields the following assertions.

When the tangential stress at the boundary τ_1 is given, a critical value of applied tangential stress τ_{cr} and the corresponding critical value of the parameter σ_{cr} given by the equation

$$\sigma_{cr}\,\mathrm{th}\,\sigma_{cr} = 1 \quad (\sigma_{cr} \approx 1{,}2) \tag{1.11}$$

exist such, that Eq.(1.9) has a solution in σ only when $\tau_1 \leqslant \tau_{cr}$. When $\tau_1 < \tau_{cr}$, we have two solutions of (1.9) in σ. The smaller of these solutions $(\sigma < \sigma_{cr})$ gives a low-temperature solution of the stationary problem, and the larger solution $(\sigma > \sigma_{cr})$ is the high temperature one. When $\tau_1 = \tau_{cr}$, we have a single solution of (1.9) in σ which yields a unique solution of the stationary problem. When $\tau_1 > \tau_{cr}$, Eq.(1.9) has no solutions in σ, and this implies that the stationary problem has no solution (a hydrodynamic thermal explosion occurs).

When the velocities of the boundary planes $\pm u_1$ are given, Eq.(1.10) always has a solution which is unique in σ. This means that the solution of the stationary problem always exists (there is no hydrodynamic thermal explosion) and is unique.

2. Let us simplify the non-stationary problem by assuming that the fluid is highly viscous ($\mathrm{Pr} \gg 1$). This means that the characteristic time of flow of the fluid is much shorter than the characteristic time of temperature relaxation. Such a flow is dynamically quasi-stationary, i.e. the velocity distribution in its "rapid" time manages to adjust itself to the "slowly" varying temperature distribution. The second equation of (1.1) becomes, under this assumption, the equation $\partial\tau/\partial z = 0$, from which it follows that the tangential stress is constant over space just as in the stationary case, but varies, generally speaking, with time. When the tangential stress is given at the boundary (condition (1.3)).

(type 1) $$\tau = \tau_1 \tag{2.1}$$

and consequently the tangential stress τ does not vary with time. When the velocities of the boundary planes are given (condition (1.4)), then, averaging the third equation of (1.1) over the interval from -1 to 1, we obtain

(type 2) $$\tau \langle \varphi(\vartheta) \rangle = u_1 \tag{2.2}$$

from which we see that the tangential stress τ and temperature ϑ both vary with time.

Therefore, in the first case the first equation of (1.1), which can be solved together with the trivial relation (2.1), is reduced to

(type 1) $$\frac{\partial\vartheta}{\partial t} = \frac{\partial^2\vartheta}{\partial z^2} + 2\tau_1^2\varphi(\vartheta)$$

which describes non-stationary heat transfer in a medium with heat sources depending non-linearly on temperature. In the second case the first equation of (1.1) and relation (2.2) are solved together and reduce to a single equation

(type 2) $$\frac{\partial\vartheta}{\partial t} = \frac{\partial^2\vartheta}{\partial z^2} + 2u_1^2\varphi(\vartheta)/\langle\varphi(\vartheta)\rangle$$

which can also be regarded as one describing heat transfer in the medium containing heat sources depending non-linearly on temperature, with one difference. The heat source intensity at each point depends on temperature not only at this point, but also on the temperature distribution over the whole volume.

Let us investigate the stability of the stationary solutions under small perturbations. We shall write the non-stationary quantities ϑ, τ and u in the form of a sum of stationary solutions $\bar\vartheta, \bar\tau$ and $\bar u$, and small perturbations ϑ', τ' and u' (henceforth we shall omit the primes). Let us linearize the first equation of (1.1) and relations (2.1) and (2.2) with respect to small perturbations, assuming that the perturbations vary exponentially with time with the increment λ, and using for the stationary solutions the first equation of (1.5) and relations (1.6), (1.7). As a result we obtain the following equation for the perturbations:

$$\frac{d^2\vartheta}{dz^2} + 2\bar\tau^2\varphi'(\bar\vartheta)\vartheta + 4\bar\tau\varphi(\bar\vartheta)\tau = \lambda\vartheta \tag{2.3}$$

with boundary condition (1.2) and relations

(type 1) $$\tau = 0 \tag{2.4}$$

(type 2) $$\bar\tau\langle\varphi'(\bar\vartheta)\vartheta\rangle \quad \langle\varphi(\bar\vartheta)\rangle\tau = 0 \tag{2.5}$$

Reducing Eq.(2.3) and relations (2.4) or (2.5) to a single equation, we obtain

(type 1) $$\frac{d^2\vartheta}{dz^2} + 2\bar\tau^2\varphi'(\bar\vartheta)\vartheta = \lambda\vartheta$$

(type 2) $$\frac{d^2\vartheta}{dz^2} + 2\bar\tau^2\varphi'(\bar\vartheta)\vartheta - 4\bar\tau^2\varphi(\bar\vartheta)\langle\varphi'(\bar\vartheta)\vartheta\rangle/\langle\varphi(\bar\vartheta)\rangle = \lambda\varphi \tag{2.6}$$

The boundary conditions have, as before, the form (1.2).

Thus in the first case we have the standard Sturm-Liouville problem, which cannot be said of the second case. In both cases the corresponding Eq.(2.6) with boundary condition (1.2) forms a selfconjugate problem. Therefore the incremental spectrum is real and there is no oscillatory mode.

3. In what follows, we shall consider the case of exponential fluidity, using formulas (1.8) for the stationary solutions. It will now be convenient (/20/, Sect.23, problem 5) to change to a new variable $\zeta = \operatorname{th}(\sigma z)$ and write $\lambda = \mu^2\sigma^2$. Then Eqs.(2.6) will be written in the form (type 1) $$\frac{d}{d\zeta}\left((1-\zeta^2)\frac{d\vartheta}{d\zeta}\right)+\left(2-\frac{\mu^2}{1-\zeta^2}\right)\vartheta=0 \tag{3.1}$$

(type 2) $$\frac{d}{d\zeta}\left((1-\zeta^2)\frac{d\vartheta}{d\zeta}\right)+\left(2-\frac{\mu^2}{1-\zeta^2}\right)\vartheta=4\langle\vartheta\rangle \tag{3.2}$$

Here $\langle\ldots\rangle$ denotes averaging over the interval from $-\omega$ to ω, where $\omega = \operatorname{th}\sigma$. The boundary conditions in the new variables will have the form

$$\zeta = \pm\omega,\ \vartheta = 0 \tag{3.3}$$

We note, before anything else, that when $\sigma = 0$, both equations of (2.6) degenerate into the equation $d^2\vartheta/dz^2 = 0$, and this implies that the incremental spectrum is negative, i.e. when $\sigma = 0$, the stationary solutions are stable.

We can expect that the limit of stability will be those $\sigma = \sigma_0$ for which the maximum increment will vanish. Therefore, to determine the limits of stability, we write in Eqs.(3.1) and (3.2) $\mu = 0$ and find $\sigma = \sigma_0$ for which solutions of these equations exist satisfying the boundary conditions (3.3).

When $\mu = 0$, the solutions of (3.1) are Legendre functions of order 1 /21/, i.e.

(type 1) $$\vartheta = \alpha\zeta + \beta(\zeta\operatorname{Arth}\zeta - 1),\ \alpha,\beta = \text{const}$$

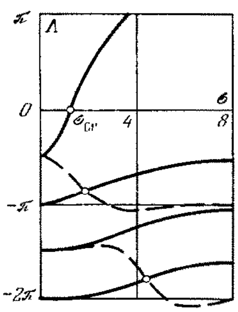
From the boundary conditions (3.3) it follows that $\alpha = 0$ and $\sigma_0 \operatorname{th}\sigma_0 = 1$. The last equation is the same as (1.11), i.e. the limit of stability σ_0 is the same as the critical value σ_{cr}.

Thus when $\sigma = 0$, all increments are negative and they remain such when $\sigma < \sigma_{cr}$; when $\sigma = \sigma_{cr}$, one of the increments, which is necessarily the largest one, vanishes; when $\sigma > \sigma_{cr}$, the largest increment becomes positive and there are no other passages through zero. (Some of the above conclusions hold only when the spectral curves are situated in a general position. The results of Sect.4 show that this is indeed the case). In other words, when $\sigma < \sigma_{cr}$ we have stability, when $\sigma = \sigma_{cr}$ we have neutral stability and when $\sigma > \sigma_{cr}$ we have instability. Therefore, in the case when the tangential stress is specified at the boundary,or, which amounts to the same, if heat is transferred in the medium with heat sources which depend non-linearly but locally and also exponentially, on temperature, then out of the two stationary solutions, the low temperature one is stable and the high temperature one unstable, which agrees with the results of /1–6/.

When $\mu = 0$, we can obtain the solution of (3.2) from solutions of (3.1) with $\mu = 0$, using the method of varying the constants. We have $\vartheta = \alpha\zeta + \beta(\zeta\operatorname{Arth}\zeta - 1) + 2\langle\vartheta\rangle$. Taking the mean, we finally obtain

(type 2) $$\vartheta = \alpha\zeta + \beta(\zeta\operatorname{Arth}\zeta - 1 + f(\sigma)),\ \alpha,\beta = \text{const}$$
$$f(\sigma) = (\sigma + \operatorname{th}\sigma - \sigma\operatorname{th}^2\sigma)/\operatorname{th}\sigma$$

From the boundary conditions (3.3) it follows that $\alpha = 0$ and $\sigma_0\operatorname{th}\sigma_0 = 1 - f(\sigma_0)$. The last equation has no solutions. This means that the increments will not pass through zero no matter what the value of σ, and since they are negative when $\sigma = 0$, they will remain negative for all values of σ. This implies that when the velocities of the boundary planes are given, the unique stationary solution is always stable.



4. Let us find the spectrum of the perturbation increments for an arbitrary value of the parameter σ. Before anything else, we note that Eqs.(3.1) and (3.2) together with the boundary conditions (3.3) form selfconjugate problems and are invariant under reflection $\zeta \to -\zeta$. Therefore, the incremental spectrum is real and the eigenfunctions can, for at least a single (non-repeated) spectrum, be either even or odd.

In the first case when we must start with Eq.(3.1), its solutions are the generalized Legendre functions of the order of unity and degree μ /21/. Combining these functions linearly, we obtain the following pair of even solutions a_+ and odd solutions a_- of (3.1):

$$a_+(\zeta) = \operatorname{ch}(\mu\operatorname{Arth}\zeta) - \zeta\operatorname{sh}(\mu\operatorname{Arth}\zeta)/\mu$$
$$a_-(\zeta) = \zeta\operatorname{ch}(\mu\operatorname{Arth}\zeta) - \mu\operatorname{sh}(\mu\operatorname{Arth}\zeta)$$

We seek the spectrum from the requirement that boundary conditions (3.3) hold for the solutions $a_\pm$. The general pattern of behaviour of the spectrum (the quantities $\Lambda = \operatorname{sign}\lambda\sqrt{|\lambda|}$) relative to σ is shown in the figure with solid lines.

In the second case, in which we must start with Eq.(3.2), its even solution b_+ can be obtained from the solutions $a_\pm$ of (3.1) and by using the method of varying the constants. From the boundary conditions (3.3) we obtain

$$b_+(\zeta) = \frac{4\langle b_+\rangle}{(1-\mu^2)\,a_+(\omega)}\left(a_+(\omega)\,a(\zeta) - a_+(\zeta)\,a(\omega)\right)$$

$$a(\zeta) = a_-(\zeta) A_+(\zeta) - a_+(\zeta) A_-(\zeta), \quad A_\pm(\zeta) = \int_0^\zeta a_\pm(\zeta')\, d\zeta'$$

Taking the mean, we arrive at the equation

$$a_+(\omega) \int_0^\omega a(\zeta)\, d\zeta - a(\omega) \int_0^\omega a_+(\zeta)\, d\zeta - \omega(1-\mu^2)\, a_+(\omega) = 0$$

The problem of the behaviour of the increments with even indices relative to σ is shown in the figure by dashed lines.

For the odd solution of (3.2) we have $b_- = a_-$, since the mean of an odd function is zero. It follows from this that the increments with odd indices from the previous case will also be the increments in this case.

The main special feature characterizing the behaviour of the incremental spectrum is the fact that as σ increases, i.e. as the velocity of one boundary relative to the other boundary increases, the perturbation increments change places pairwise (an increment with the even index and the increment with the next higher odd index) during the passage through some value of σ specific for every pair, and the first pair of increments changes places during the passage through $\sigma \approx 1{,}8$. This implies that the character of the relaxation of the perturbations changes during the passage through such values of σ beginning with $\sigma \approx 1{,}8$. Moreover, all increments are separated uniformly (for all values of σ) from zero by an amount $\pi^2/4$. This means that irrespective of the value of the relative velocities of the boundary planes, the long wave (with small wave numbers) two- and three-dimensional perturbations, as well as the one-dimensional perturbations discussed here, cannot upset the stability of the stationary mode.

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ONE REPRESENTATION OF THE CONDITIONS OF THE COMPATIBILITY OF DEFORMATIONS*

V.I. MALYI

It is shown that the conditions of the compatibility of deformations can be represented in the form of three equations in the region occupied by the deformed body, and three boundary conditions on its surface. A combination of the requirements of the conditions of equilibrium and campatibility leads to a unique formulation of the problem in terms of the stresses for the deformed body in the form of a system of six equations for the six unknown components of the stress tensor, and of a set of boundary conditions corresponding to the ninth order of the system of equations.

The classical formulation of the problem in terms of the stresses for a deformed rigid body leads to the need to solve a system containing its three equations of equilibrium and six compatability equations for six unknown components of the stress tensor. It can therefore be expected that some of the demands imposed by the formulation of the problem may be redundant. After all, such reasoning has been used systematically in similar situations in the scientific literature when formulating new problems, and was found to be effective.

1. We shall consider an elastic body occupying a three-dimensional region V, bounded by the surface S. We introduce in the region a Cartesian system of coordinates x_i with basis vector $\mathbf{e}_i$, so that the vector $\mathbf{n}$ normal to the surface S has components n_i. We shall denote differentiation with respect to the x_i coordinate by the index following the comma. We assume that the volume forces f_i and surface forces F_i are given. The mechanical properties of the material of the body in question will be described, generally speaking, by the following non-linear defining relations:

$$\varepsilon_{ij} = \varepsilon_{ij}(\sigma_{kl}) \tag{1.1}$$

connecting the deformation tensor ε_{ij} and stress tensor σ_{kl}.

The classical formulation of the boundary value problem of the mechanics of a deformable rigid body in terms of the stresses has certain specific features which merit attention. It is insufficient to satisfy three equations of equilibrium

$$\sigma_{ij,j} + f_i = 0, \quad \mathbf{x} \in V \tag{1.2}$$

with static boundary conditions

$$\sigma_{ij} n_j = F_i, \quad \mathbf{x} \in S \tag{1.3}$$

in order to determine uniquely the six components of the stress tensor σ_{ij}. Since the stresses are connected with the deformations by means of the defining Eqs.(1.1), the missing relations can be obtained from the natural geometrical Saint Venant conditions of compatibility for the components $\varepsilon_{ij}(\mathbf{x})$ of the deformation tensor. These can be written in the form /1/

$$g_{ij}(\mathbf{x}) \equiv \varepsilon_{ij,kk} + \varepsilon_{kk,ij} - \varepsilon_{ik,kj} - \varepsilon_{jk,ki}, \quad \mathbf{x} \in V \tag{1.4}$$

Although now we have more relations (1.4) than is necessary to formulate a definite system of equations for six functions σ_{ij} (or ε_{ij}) and the system (1.1) becomes overdefined

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